

NEW RANKING OF TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS BASED ON THE SCORE FUNCTION

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ABSTRACT

In this paper, a new method for ranking trapezoidal intuitionistic fuzzy numbers is proposed. The proposed method considers the areas on the positive side, the areas on the negative side of membership function and nonmembership function of trapezoidal intuitionistic fuzzy numbers to evaluate ranking scores of the trapezoidal intuitionistic fuzzy numbers.

KEYWORDS: Fuzzy Number, Intuitionistic Fuzzy Number, Trapezoidal Fuzzy Number, Trapezoidal Intuitionistic Fuzzy Number

1. INTRODUCTION

Zadeh [1] introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life. The concept of Intuitionistic fuzzy set [4,6] can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. The intuitionistic fuzzy sets were first introduced by Atanassov [4] which is a generalization of the concept of fuzzy set [1]. Ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. Fuzzy numbers must be ranked before an action is taken by a decision maker.

Real numbers can be linearly ordered by the relation \leq or \geq , however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient method for ordering the fuzzy numbers is by the use of a ranking function, which maps each fuzzy number into the real line, where a natural order exists.

The concept of ranking function for comparing normal fuzzy numbers is compared in Jain [2]. Abbasbandy, Hajjari [11] presented a new approach for ranking of trapezoidal fuzzy numbers. Shyi-Ming Chen, Kata Sanguansat [13] proposed a new fuzzy ranking method between generalized fuzzy numbers. In Mitchell [9] and Nayagam [10], some methods for ranking of intuitionistic fuzzy numbers were introduced. Grzegrorzewski [8] suggested the method of ranking Intuitionistic fuzzy numbers and an ordering method for Intuitionistic fuzzy number by using the expected interval of an Intuitionistic fuzzy number. Based on the characteristic value for a fuzzy number introduced in Kuo-Ping Chiao [7], an ordering method for Intuitionistic fuzzy number is proposed by Hassan Mishmast Nehi [12].

In this paper, a new method for ranking trapezoidal intuitionistic fuzzy numbers is proposed. The proposed method considers the areas on the positive side, the areas on the negative side of membership function and nonmembership function of trapezoidal intuitionistic fuzzy numbers to evaluate ranking scores of the trapezoidal intuitionistic fuzzy numbers.

2. PRELIMINARIES ([3],[4],[5],[6],[7],[12])

Definition 2.1: If *X* is a collection of objects denoted generically by *x*, then a fuzzy set \tilde{A} in *X* is defined to be a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set.

The membership function maps each element of X to a membership value between 0 and 1.

Remark: We assume that *X* is the real line *R*

Definition 2.2: A fuzzy set \tilde{A} of the real line *R* with the membership function $\mu_{\tilde{A}}: R \longrightarrow [0, 1]$ is called a fuzzy number if

- \tilde{A} is normal. ie. There exist an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$
- \tilde{A} is fuzzy convex.

ie. $\forall x_1, x_2 \in R, \forall \lambda \in [0,1], \mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge \mu_{\tilde{A}}(x_1) \land \mu_{\tilde{A}}(x_2)$

- $\mu_{\tilde{A}}$ is upper semi continuous.
- \tilde{A} is bounded.

Definition 2.3: Let X be the universal set. An Intuitionistic fuzzy set (IFS) \tilde{A} in X is given by

 $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) / x \in X\} \text{ where the functions } \mu_{\tilde{A}}, \nu_{\tilde{A}} \colon X \longrightarrow [0, 1] \text{ are functions such that} \\ 0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1 \forall x \in X.$

For each *x* the numbers $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of the element *x* $\in X$ to the set, which is a subset of *X*, respectively.

Definition 2.4: For each IFS $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)) | x \in X\}$ in X, we will call

 $\prod_{\tilde{A}} = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) \text{ as the Intuitionistic fuzzy index of } X \text{ in } \tilde{A}. \text{ It is obvious that } 0 \le \Pi_{\tilde{A}} \le 1 \forall x \in X.$

Definition 2.5: An IFS $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) | x \in X\}$ is called IF- normal, if there exists at least two points $x_0, x_1 \in X$ such that $\mu_{\tilde{A}}(x_0) = 1, \nu_{\tilde{A}}(x_1) = 1$.

Definition 2.6: An IFS $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) | x \in X\}$ of the real line is called IF- convex if $\forall x_1, x_2 \in R, \forall \lambda \in [0,1], \mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \ge \mu_{\tilde{A}}(x_1) \land \mu_{\tilde{A}}(x_2)$ and

 $v_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \ge v_{\tilde{A}}(x_1) \wedge v_{\tilde{A}}(x_2)$. Thus \tilde{A} is IF-convex if its membership function is fuzzy convex and its non membership function is fuzzy concave.

Definition 2.7: An IFS $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) | x \in X\}$ of the real line is called an Intuitionistic fuzzy number (IFN) if (a) \tilde{A} is IF-normal, (b) \tilde{A} is IF-convex, (c) $\mu_{\tilde{A}}$ is upper semi continuous and $\nu_{\tilde{A}}$ is lower semi continuous.

(d) $\tilde{A} = \{x \in X / \nu_{\tilde{A}}(x) < 1\}$ is bounded.



Figure 1: Intuitionistic Fuzzy Number

For any IFN \tilde{A} there exists eight numbers a_1 , a_2 , a_3 , a_4 , b_1 , b_2 , b_3 , $b_4 \in R$ such that

 $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ and four functions $f_{\tilde{A}}, g_{\tilde{A}}, h_{\tilde{A}}, k_{\tilde{A}}$: $R \longrightarrow [0,1]$, called the sides of a fuzzy number, where $f_{\tilde{A}}$ and $k_{\tilde{A}}$ are nondecreasing and $g_{\tilde{A}}$ and $h_{\tilde{A}}$ are nonincreasing, such that we can describe a membership function $\mu_{\tilde{A}}$ and non membership $\nu_{\tilde{A}}$ in the following form

In particular if the decreasing functions $g_{\tilde{A}}$ and $h_{\tilde{A}}$ and increasing functions $f_{\tilde{A}}$ and $k_{\tilde{A}}$ are linear then we will have the trapezoidal intuitionistic fuzzy numbers (TIFN).

Definition 2.8: \tilde{A} is a trapezoidal intuitionistic fuzzy number with parameters

$$b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$$
 and denoted by $\bar{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$

In this case,

$$\mu_{\bar{A}}(x) = \begin{cases} 0 & \text{if } x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ \frac{x-a_4}{a_3-a_4} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{if } a_4 < x \end{cases} \text{ and } \nu_{\bar{A}}(x) = \begin{cases} 0 & \text{if } x < b_1 \\ \frac{x-b_1}{b_2-b_1} & \text{if } b_1 \le x \le b_2 \\ 0 & \text{if } b_2 \le x \le b_3 \\ \frac{x-b_4}{b_3-b_4} & \text{if } b_3 \le x \le b_4 \\ 0 & \text{if } b_4 < x \end{cases}$$

Figure 2: Trapezoidal Intuitionistic Fuzzy Number

If in a TIFN \tilde{A} , we let $b_2 = b_3$ (and hence $a_2 = a_3$) then we will give a Triangular Intuitionistic fuzzy number (TrIFN) with parameters $b_1 \le a_1 \le b_2$ ($a_2 = a_3 = b_3$) $\le a_4 \le b_4$ and denoted by $\tilde{A} = (b_1, a_1, b_2, a_4, b_4)$.

3. PROPOSED RANKING METHOD FOR TIFN

In this section we present a new method for ranking trapezoidal intuitionistic fuzzy numbers. The proposed method calculates the areas on the positive side, the areas on the negative side of the membership function and nonmembership function of trapezoidal intuitionistic fuzzy numbers to evaluate the ranking scores of the trapezoidal intuitionistic fuzzy numbers. Assume that there are n trapezoidal intuitionistic fuzzy numbers $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n$ to be ranked, where

 $\tilde{A}_i = (b_{i1}, a_{i1}, b_{i2}, a_{i2}, a_{i3}, b_{i3}, a_{i4}, b_{i4}), \quad -\infty \le b_{i1} \le a_{i1} \le b_{i2} \le a_{i2} \le a_{i3} \le b_{i3} \le a_{i4} \le b_{i4} \le \infty, \quad \text{and} \quad 1 \le i \le n.$ The proposed method for ranking trapezoidal intuitionistic fuzzy numbers is shown as follows:

Step 1: Transform each trapezoidal intuitionistic fuzzy number $\tilde{A}_i = (b_{i1}, a_{i1}, b_{i2}, a_{i2}, a_{i3}, b_{i3}, a_{i4}, b_{i4})$ into a standardized trapezoidal intuitionistic fuzzy number \tilde{A}_i^* ,

$$\tilde{A}_{i}^{*} = \left(\frac{b_{i1}}{k_{1}}, \frac{a_{i1}}{k}, \frac{b_{i2}}{k}, \frac{a_{i2}}{k}, \frac{a_{i3}}{k}, \frac{b_{i3}}{k_{1}}, \frac{a_{i4}}{k}, \frac{b_{i4}}{k_{1}}\right) = \left(b_{i1}^{*}, a_{i1}^{*}, b_{i2}^{*}, a_{i3}^{*}, b_{i3}^{*}, a_{i4}^{*}, b_{i4}^{*}\right)$$
(1)

where $k = max_{ij}$ ([$|a_{ij}|$], 1), $|a_{ij}|$ denotes the absolute value of a_{ij} and [$|a_{ij}|$] denotes the upper bound of $|a_{ij}|$, $1 \le i \le n$, $1 \le j \le 4$ and $k_1 = max_{ij}$ ([$|b_{ij}|$], 1), $|b_{ij}|$ denotes the absolute value of b_{ij} and [$|b_{ij}|$] denotes the upper bound of $|b_{ij}|$, $1 \le i \le n$, $1 \le j \le 4$

Step 2: Calculate the areas Area⁻_{µiL} and Area⁻_{µiR} on the negative side, respectively, which denote the trapezoidal areas from the membership function curve of the trapezoidal intuitionistic fuzzy number (-1, -1, -1, -1, -1, -1, -1, -1) to the membership function curves of $f_{\tilde{A}_i^*}$ and $g_{\tilde{A}_i^*}$ of the standardized intuitionistic fuzzy number \tilde{A}_i^* where

$$f_{\tilde{A}_{i}^{*}} = \frac{x - a_{i1}^{*}}{a_{i2}^{*} - a_{i1}^{*}}, a_{i1}^{*} \le x \le a_{i2}^{*}$$

$$\tag{2}$$

$$g_{\tilde{A}_{i}^{*}} = \frac{x - a_{i4}^{*}}{a_{i3}^{*} - a_{i4}^{*}}, a_{i3}^{*} \le x \le a_{i4}^{*}$$
(3)

and Area_{µiL}⁻ =
$$\frac{(a_{i1}^* + 1) + (a_{i2}^* + 1)}{2}$$
, Area_{µiR}⁻ = $\frac{(a_{i3}^* + 1) + (a_{i4}^* + 1)}{2}$

Then calculate the areas $\text{Area}_{\mu i L}^+$ and $\text{Area}_{\mu i R}^+$ on the positive side, respectively, which denote the trapezoidal areas from the membership function curves of $f_{A_i^*}$ and $g_{A_i^*}$ of the standardized trapezoidal intuitionistic fuzzy number \tilde{A}_i^* defined in equations (2) and (3) respectively to the membership function curve of the trapezoidal intuitionistic fuzzy number

$$(-1, -1, -1, -1, -1, -1, -1, -1, -1)$$
 where $\operatorname{Area}_{\mu i L}^{+} = \frac{(1 - a_{i1}^{*}) + (1 - a_{i2}^{*})}{2} \operatorname{Area}_{\mu i R}^{+} = \frac{(1 - a_{i3}^{*}) + (1 - a_{i4}^{*})}{2}$

The areas Area⁻_{µiL}, Area⁺_{µiR}, Area⁺_{µiR}, are shown as the shaded reginons in Figure 3 (a), (b), (c), (d) respectively.



Figure 3: (a) Area $^{-}_{\mu i L}$ (b) Area $^{-}_{\mu i R}$ (c) Area $^{+}_{\mu i L}$ (d) Area $^{+}_{\mu i R}$

Step 3: Calculate the values $XI_{\tilde{A}_{\mu i}^*}$ and $XD_{\tilde{A}_{\mu i}^*}$ of each standardized trapezoidal intuitionistic fuzzy number \tilde{A}_i^* where $XI_{\tilde{A}_{\mu i}^*}$ denotes the sum of the factors on the X- axis of $\tilde{A}_{\mu i}^*$ that have a positive infuence on the ranking score of $\tilde{A}_{\mu i}^*$, that is the ranking score increases as the values of these factors increase and $XD_{\tilde{A}_{\mu i}^*}$ denotes the sum of the factors on the X- axis of $\tilde{A}_{\mu i}^*$ that have a negative infuence on the ranking score of $\tilde{A}_{\mu i}^*$, that is the ranking score decreases as the values of these factors increase of $\tilde{A}_{\mu i}^*$, that is the ranking score decreases as the values of these factors increase shown as follows:

$$XI_{\tilde{A}_{\mu i}^{*}} = \text{Area}_{\mu i L}^{-} + \text{Area}_{\mu i R}^{-}, XD_{\tilde{A}_{\mu i}^{*}} = \text{Area}_{\mu i L}^{+} + Area_{\mu i R}^{+}$$

Step 4: Calculate the ranking score $Score(\tilde{A}_{\mu i}^*)$ of membership function of each standardized intuitionistic fuzzy number \tilde{A}_i^* shown as follows:

$$Score(\tilde{A}_{\mu i}^{*}) = \frac{1 \times XI_{\tilde{A}_{\mu i}^{*}} + (-1) \times XD_{\tilde{A}_{\mu i}^{*}}}{XI_{\tilde{A}_{\mu i}^{*}} + XD_{\tilde{A}_{\mu i}^{*}}} = \frac{XI_{\tilde{A}_{\mu i}^{*}} - XD_{\tilde{A}_{\mu i}^{*}}}{XI_{\tilde{A}_{\mu i}^{*}} + XD_{\tilde{A}_{\mu i}^{*}}}$$
(4)

where $Score(\tilde{A}_{\mu i}^*) \in [-1,1]$, $1 \le i \le n$. The larger the value of $Score(\tilde{A}_{\mu i}^*)$, the better the ranking of $\tilde{A}_{\mu i}^*$. Based on the equation (4) we can see that $XI_{\tilde{A}_{\mu i}^*}$ and $XD_{\tilde{A}_{\mu i}^*}$ are used for weighting the maximal and the minimal possible values of the universe of discourse of $\tilde{A}_{\mu i}^*$, 1 and -1 respectively. This means the larger the value of $XI_{\tilde{A}_{\mu i}^*}$, the closer the value of $Score(\tilde{A}_{\mu i}^*)$ is to 1; the larger the value of $XD_{\tilde{A}_{\mu i}^*}$, the closer the value of $Score(\tilde{A}_{\mu i}^*)$ is to -1.

Step 5: If the $Score(\tilde{A}_{\mu i}^*)$, $1 \le i \le n$ of membership function of each standardized trapezoidal intuitionistic fuzzy number \tilde{A}_i^* are all equal then calculate the $Score(\tilde{A}_{\gamma i}^*)$, $1 \le i \le n$ of nonmembership function of each standardized trapezoidal intuitionistic fuzzy number \tilde{A}_i^* as follows

Step 6: Calculate the areas Area $_{\gamma iL}$ and Area $_{\gamma iR}^-$ on the negative side, respectively, which denote the trapezoidal areas from the nonmembership function curve of the trapezoidal intuitionistic fuzzy number (-1, -1, -1, -1, -1, -1, -1, -1) to the nonmembership function curves of $h_{\tilde{A}_i^*}$ and $k_{\tilde{A}_i^*}$ of the standardized trapezoidal intuitionistic fuzzy number \tilde{A}_i^* where

$$h_{\tilde{A}_{i}^{*}} = \frac{x - b_{i1}^{*}}{b_{i2}^{*} - b_{i1}^{*}}, b_{i1}^{*} \le x \le b_{i2}^{*}$$
(5)

$$k_{\tilde{A}_{i}^{*}} = \frac{x - b_{i4}^{*}}{b_{i3}^{*} - b_{i4}^{*}}, b_{i3}^{*} \le x \le b_{i4}^{*}$$
(6)

and Area<sub>$$\gamma$$
iL</sub> = $\frac{(b_{i1}^* + 1) + (b_{i2}^* + 1)}{2}$ Area _{γ iR} = $\frac{(b_{i3}^* + 1) + (b_{i4}^* + 1)}{2}$

Then calculate the areas $\operatorname{Area}_{\gamma i \mathsf{L}}^+$ and $\operatorname{Area}_{\gamma i \mathsf{R}}^+$ on the positive side, respectively, which denote the trapezoidal areas from the nonmembership function curves of $h_{\tilde{A}_i^*}$ and $k_{\tilde{A}_i^*}$ of the standardized trapezoidal intuitionistic fuzzy number \tilde{A}_i^* defined in equations (5) and (6) respectively to the nonmembership function curve of the trapezoidal intuitionistic fuzzy number

$$(-1, -1, -1, -1, -1, -1, -1, -1, -1)$$
 where $\operatorname{Area}_{\gamma i L}^{+} = \frac{(1 - b_{i1}^{*}) + (1 - b_{i2}^{*})}{2}$ and $\operatorname{Area}_{\gamma i R}^{+} = \frac{(1 - b_{i3}^{*}) + (1 - b_{i4}^{*})}{2}$

The areas Area⁻_{$\gamma iL}, Area⁺_{<math>\gamma iR}$, Area⁺_{$\gamma iR},$ *Area* $⁺_{<math>\gamma iR}$ are shown as the shaded reginons in Figure 4 (a), (b), (c), (d) respectively.</sub></sub></sub></sub>



Figure 4: (a) Area $_{\gamma iL}^{-}$ (b) Area $_{\gamma iR}^{-}$, (c) Area $_{\gamma iL}^{+}$, (d) Area $_{\gamma iR}^{+}$

Step 7: Calculate the values $XI_{\tilde{A}_{\gamma i}^*}$ and $XD_{\tilde{A}_{\gamma i}^*}$ of each standardized trapezoidal intuitionistic fuzzy number \tilde{A}_i^* where $XI_{\tilde{A}_{\gamma i}^*}$ denotes the sum of the factors on the X- axis of $\tilde{A}_{\gamma i}^*$ that have a positive infuence on the ranking score of $\tilde{A}_{\gamma i}^*$, that is the ranking score increases as the values of these factors increase and $XD_{\tilde{A}_{\gamma i}^*}$ denotes the sum of the factors on the X- axis of $\tilde{A}_{\gamma i}^*$ that have a negative infuence on the ranking score of $\tilde{A}_{\gamma i}^*$, that is the ranking score decreases as the values of these factors increase of $\tilde{A}_{\gamma i}^*$, that is the ranking score decreases as the values of these factors increase shown as follows:

$$XI_{\tilde{A}_{\gamma i}^{*}} = \operatorname{Area}_{\gamma i L}^{-} + \operatorname{Area}_{\gamma i R}^{-}, XD_{\tilde{A}_{\gamma i}^{*}} = \operatorname{Area}_{\gamma i L}^{+} + \operatorname{Area}_{\gamma i R}^{+}$$

Step 8: Calculate the ranking score $Score(\tilde{A}^*_{\gamma i})$ of nonmembership function of each standardized intuitionistic fuzzy number \tilde{A}^*_i shown as follows:

$$Score(\tilde{A}_{\gamma i}^{*}) = \frac{1 \times XI_{\tilde{A}_{\gamma i}^{*}} + (-1) \times XD_{\tilde{A}_{\gamma i}^{*}}}{XI_{\tilde{A}_{\gamma i}^{*}} + XD_{\tilde{A}_{\gamma i}^{*}}} = \frac{XI_{\tilde{A}_{\gamma i}^{*}} - XD_{\tilde{A}_{\gamma i}^{*}}}{XI_{\tilde{A}_{\gamma i}^{*}} + XD_{\tilde{A}_{\gamma i}^{*}}}$$
(7)

where $Score(\tilde{A}_{\gamma i}^*) \in [-1,1]$, $1 \le i \le n$. The larger the value of $Score(\tilde{A}_{\gamma i}^*)$, the better the ranking of \tilde{A}_i^* . Based on the equation (7) we can see that $XI_{\tilde{A}_{\gamma i}^*}$ and $XD_{\tilde{A}_{\gamma i}^*}$ are used for weighting the maximal and the minimal possible values of the universe of discourse of $\tilde{A}_{\gamma i}^*$, 1 and -1 respectively. This means the larger the value of $XI_{\tilde{A}_{\gamma i}^*}$, the closer the value of $Score(\tilde{A}_{\gamma i}^*)$ is to 1; the larger the value of $XD_{\tilde{A}_{\gamma i}^*}$, the closer the value of $Score(\tilde{A}_{\gamma i}^*)$ is to -1.

Step 9: If the $Score(\tilde{A}^*_{\gamma i})$, $1 \le i \le n$ of nonmembership function of each standardized trapezoidal intuitionistic fuzzy number \tilde{A}^*_i are all equal then all \tilde{A}_i are equal.

CONCLUSIONS

In this paper, we have found the standardized trapezoidal intuitionistic fuzzy number, areas on the negative side, positive side of membership function and non-membership function of a trapezoidal intuitionistic fuzzy numbers and proposed a new method for ranking of trapezoidal Intuitionistic fuzzy numbers based on the scoring. The proposed method provides the exact ordering of trapezoidal intuitionistic fuzzy numbers. This approach can be applied to rank the trapezoidal intuitionistic fuzzy numbers in solving different intuitionistic fuzzy optimization problems.

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